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Predictions for the frequency and orbital radii of massive extrasolar planets

Philip J. Armitage[1,2], Mario Livio[3], S.H. Lubow[3] and J.E. Pringle[4,3]
[1] *JILA, University of Colorado, 440 UCB, Boulder CO 80309-0440, USA*
[2] *School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews KY16 9SS, UK*
[3] *Space Telescope Science Institute, 3700 San Martin Drive, Baltimore MD 21218, USA*
[4] *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK*

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ABSTRACT
We investigate the migration of massive extrasolar planets due to gravitational interaction with a viscous protoplanetary disc. We show that a model in which planets form at 5 AU at a constant rate, before migrating, leads to a predicted distribution of planets that is a steeply rising function of $\log(a)$, where a is the orbital radius. Between 1 AU and 3 AU, the expected number of planets per logarithmic interval in a roughly doubles. We demonstrate that, once selection effects are accounted for, this is consistent with current data, and then extrapolate the observed planet fraction to masses and radii that are inaccessible to current observations. In total, about 15% of stars targeted by existing radial velocity searches are predicted to possess planets with masses $0.3M_{\rm J} < M_p \sin(i) < 10M_{\rm J}$, and radii $0.1{\rm AU} < a < 5{\rm AU}$. A third of these planets (around 5% of the target stars) lie at the radii most amenable to detection via microlensing. A further 5-10% of stars could have planets at radii of $5{\rm AU} < a < 8{\rm AU}$ that have migrated outwards. We discuss the probability of forming a system (akin to the Solar System) in which significant radial migration of the most massive planet does *not* occur. About 10-15% of systems with a surviving massive planet are estimated to fall into this class. Finally, we note that a smaller fraction of low mass planets than high mass planets is expected to survive without being consumed by the star. The initial mass function for planets is thus predicted to rise more steeply towards small masses than the observed mass function.

Key words: accretion, accretion discs — solar system: formation — planetary systems: formation — planetary systems: protoplanetary discs — gravitational lensing

1 INTRODUCTION

Radial velocity surveys of nearby stars show that a significant fraction – at least 8% – have massive planets with orbital radii substantially less than that of Jupiter (Marcy & Butler 2000; Udry et al. 2000; Butler et al. 2001). Most of these planets lie at radii where massive planet formation is theoretically believed to be difficult (Bodenheimer, Hubickyj & Lissauer 2000). The difficulties are most pronounced for those planets at the smallest orbital radii, where the temperatures in the protoplanetary disc would have exceeded those for which ices (and possibly even dust) can exist. These problems can be avoided if planets formed at larger radii, and then migrated inwards to where they are detected today. Mechanisms that could lead to this migration include gravitational interaction with a gaseous and viscous protoplanetary disc (Lin, Bodenheimer & Richardson 1996), planet-planet scattering (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Ford, Havlickova & Rasio 2001; Papaloizou & Terquem 2001), or scattering of planetesimals by massive planets (Murray et al. 1998).

In the right circumstances, all of the suggested mechanisms can lead to substantial migration. More quantitative comparisons with the data are therefore warranted. Recently, Trilling, Lunine & Benz (2002) have reported promising results for the protoplanetary disc migration model. They showed that the broad distribution of orbital radii was consistent with the planets becoming stranded, during their inward migration, by the dispersal of the protoplanetary disc. They used the model to estimate both the fraction of stars that must have formed planets, and how many ought to have survived migration.

In this paper, we extend the work of Trilling et al. (2002). We include a physical mechanism for disc dispersal into a model for the formation and migration of massive planets, and make quantitative comparison with the observed distribution of planetary orbital radii. Our disc

model, described in Section 2, combines viscous evolution with mass loss at the outer edge, for example due to photoevaporation (Johnstone, Hollenbach & Bally 1998; Clarke, Gendrin & Sotomayor 2001). We then run this model repeatedly, on each occasion adding a single planet to the disc at a specified formation time $t_{\rm form}$. By varying $t_{\rm form}$, subject to the constraint that no planets can be formed once the disc mass is too low, we study in Section 3 how the final orbital radii of planets depend upon the formation time. With the addition of plausible assumptions about how the rate of planet formation varies with time, we then convert the results into a prediction for the radial distribution of planetary orbits. In Section 4 we compare this distribution with the current data, and find that good agreement is obtained. In Section 5 we make use of the model to estimate the fraction of stars targeted by radial velocity surveys that ought to harbour planets, including those with masses too low, or orbital radii too large, to be currently detected. We also predict the number of stars with planets at radii suitable for detection via microlensing, and the fraction of systems where significant planetary migration does not occur. Our conclusions are summarized in Section 6.

2 METHODS

2.1 Fully viscous disc model

The simplest protoplanetary disc model that meets the basic observational constraints on disc mass, disc lifetime, and disc destruction time-scale is a viscous disc with mass loss from the outer regions (Hollenbach, Yorke & Johnstone 2000; Clarke, Gendrin & Sotomayor 2001). We use this model for the majority of the calculations described in this paper, since it has a minimal number of free parameters.

For the viscous disc model runs, we assume that over the relevant range of radii (roughly, between 0.1 AU and 10 AU), the kinematic viscosity ν can be described as a power law in radius R,

$$\nu = \nu_0 \left(\frac{R}{1 \ {\rm AU}} \right)^{\beta} . \quad (1)$$

The initial conditions for the surface density are a constant accretion rate disc, with a profile which corresponds to the assumed viscosity of equation (1). At radii $R \gg R_{\rm in}$, the inner edge of the disc, this implies,

$$\Sigma = \frac{\dot{M}}{3 \pi \nu} = \Sigma_0 R^{-\beta} \quad (2)$$

where $\dot{M}$ is the accretion rate and Σ_0 a constant. For our standard disc model, we take $\beta = 3/2$ (Weidenschilling 1977; Hayashi 1981), and choose $\nu_0 = 1.75 \times 10^{13}$ cm^{2}s^{-1} to give a sensible time-scale for disc evolution of a few Myr. Finally, Σ_0 is set in the initial conditions so that the initial disc mass is $0.1 M_\odot$. The inner and outer boundaries for the calculation are $R_{\rm in} = 0.066$ AU, and $R_{\rm out} = 33.3$ AU. To extend the predictions to smaller radii, a limited number of runs have also been completed with an inner boundary at $R_{\rm in} = 0.03$ AU. A zero torque boundary condition (i.e. a surface density $\Sigma = 0$) is applied at the inner boundary, while at the outer boundary we set the radial velocity to zero.

Current observations do not directly constrain the surface density profile of protoplanetary discs at the AU scale. To check how sensitive our results are to changes in this profile, we have also run a model with $\beta = 1/2$, again normalized to give a sensible time-scale for disc evolution. A flatter profile than the $R^{-3/2}$ of our standard model is expected if the angular momentum transport within the disc follows the Shakura & Sunyaev (1973) α-prescription, with a constant α (e.g. Bell et al. 1997).

Even a small mass of gas, trapped exterior to the orbit of a massive planet, will eventually soak up a large amount of angular momentum and drive migration. This gas has to be lost in order to obtain a converged final radius for migrating planets. Trilling et al. (2002) make the simplest assumption, and halt migration by removing the disc instantaneously. We adopt a different approach, and assume that the gas is removed by photoevaporation. In this process, the surfaces of the disc are heated by the absorption of ultraviolet radiation, which may originate either from the central star or from an external source (e.g. a massive star in a young cluster). The hot gas can escape as a wind at radii $R > R_g$, where the escape velocity is less than the sound speed of the heated material. Observations of the Orion nebula show that this process drives mass loss at significant rates, at least in clusters containing massive stars (Bally, O'Dell & McCaughrean 2000).

Our implementation of photoevaporation within a time-dependent disc model is based upon the description given by Johnstone, Hollenbach & Bally (1998; see also Shu, Johnstone & Hollenbach 1993; Richling & Yorke 2000). The standard model has $R_g = 5$ AU, and a radial distribution of mass loss,

$$\begin{aligned} \dot{\Sigma} &\propto R^{-1} \quad R > R_g \\ \dot{\Sigma} &= 0 \quad R < R_g . \end{aligned} \quad (3)$$

For a disc with an outer radius $R \gg R_g$, this means that the total mass loss scales linearly with R. The initial mass loss rate (integrated over the disc out to 25 AU) is chosen to be $\dot{M} = 5 \times 10^{-9} M_\odot {\rm yr}^{-1}$.

Figure 1 shows how the disc mass changes with time, for the standard model with $\beta = 3/2$ and mass loss from $R > 5$AU. The disc mass declines from $0.1 M_\odot$ to $10^{-2} M_\odot$ in 3 Myr, mainly due to accretion, before dropping rapidly once the effects of the disc wind take hold. Although the power laws and location of the turnover vary with the adopted parameters, generically similar behavior occurs for a range of disc viscosity and mass loss rates (Clarke, Gendrin & Sotomayor 2001).

2.2 Magnetically layered disc model

In order to check the robustness of the results, we investigate how the results would change if we adopted a fundamentally different, but strongly theoretically motivated, model for disc evolution. The magnetically layered model proposed by Gammie (1996) is based upon the observation that cold gas at $T \lesssim 10^3$K has a low ionization fraction, which suppresses angular momentum transport via magnetohydrodynamic turbulence (Fleming, Stone & Hawley 2000). If there are no other sources of angular momentum transport, then only the hot inner disc, plus a surface layer ionized by cosmic

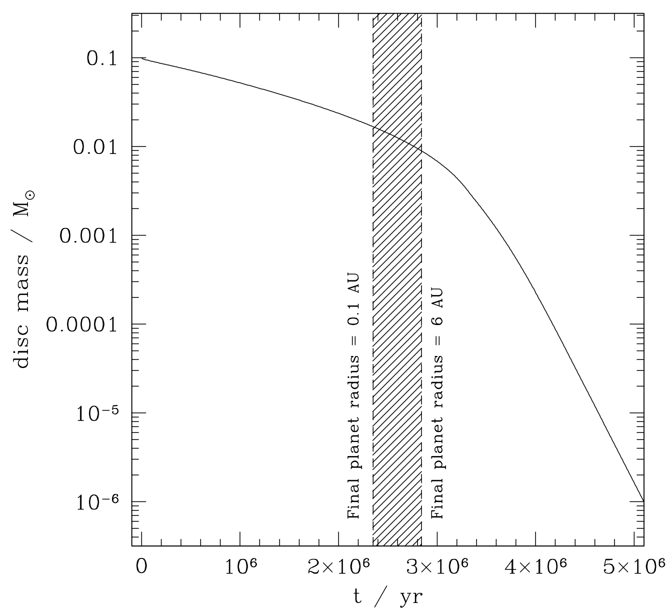

Figure 1. Evolution of the mass of the standard disc model with time. Up to around 3 Myr, the disc mass decays as gas is accreted onto the star, while the effects of the disc wind remain small. At later times, a steeper decline occurs as mass loss truncates the outer radii of the disc. The shaded band shows the range of time over which a $2M_J$ planet can be formed at 5 AU and survive with a final orbital semi-major axis $0.1\text{AU} < a < 6\text{AU}$.

rays, will be viscous. At radii $R \sim 1$AU, this implies that accretion occurs only through a thin surface layer, below which lies a thick 'dead zone' of quiescent gas. Numerical models for the long-term disc evolution (Armitage, Livio & Pringle 2001) find that there is an early phase, lasting for perhaps a Myr, during which accretion occurs in short high accretion rate outbursts. This is followed by a quiescent phase, characterised by low rates of accretion onto the star. This is due to a much reduced (compared to fully viscous disc models) viscosity at radii of the order of 1 AU.

Detailed models for the layered disc are able to provide a reasonable fit to observations of the accretion rate in Classical T Tauri stars (Gammie 1996; but see also Stepinski 1998). For the purpose of studying planetary migration, we adopt an approximate approach, and assume that the most important effect of the layered disc is to reduce the viscosity near the midplane. Specifically, after 1 Myr, we reduce the viscosity where the surface density exceeds the thickness of the magnetically active surface layer. The new viscosity $\nu_{\rm layer}$ is,

$$\nu_{\rm layer} = \nu \times \min\left(\frac{100{\rm gcm}^{-2}}{\Sigma}, 1\right) \quad (4)$$

where ν is as given in equation (1), and the same viscosity parameters as for the standard disc model are chosen. To obtain a sensible disc lifetime we also increase the mass loss rate to $\dot{M} = 3 \times 10^{-8} M_\odot {\rm yr}^{-1}$, and allow mass to be lost at all radii by taking $R_g = R_{\rm in}$.

2.3 Planet formation assumptions

The location in the disc where massive planet formation is easiest depends upon two competing factors. The characteristic time-scales for planet building in the disc are faster at small radii. However, most of the mass in protoplanetary discs lies at large radii, with a jump in the surface density of solid material beyond the 'snow line' where ices can form (Hayashi 1981; Sasselov & Lecar 2000). We assume that the outcome of this competition is that planets form at an orbital radius $a = 5$AU, similar to that of Jupiter in the Solar System. The exact choice of this location is less important than the fact that it lies outside the radii where extrasolar planets are currently observed. In the model which we are testing, this means that migration is necessary to explain the orbital radii of all currently known planets.

In each run of the disc model, we form a planet with mass M_p, at time $t_{\rm form}$ (where $t = 0$ is defined as the time when the disc mass is $0.1 M_\odot$). We specify the mass of our planets (typically $2M_J$, where M_J is the mass of Jupiter) in advance of each run, but test at the time of formation that there is enough mass in the vicinity of the planet to form it consistently. Specifically, we check that the disc mass between $R = 0.6a$ and $R = 1.6a$ exceeds the desired planet mass. Provided that this is satisfied, we remove the appropriate amount of gas from the disc around $R = a$, and add a planet with mass M_p at that location. This instantaneous formation scheme ignores the possibly lengthy time required to assemble the planet's gaseous envelope (e.g. Lissauer 1993, and references therein). However, Trilling, Lunine & Benz (2002) have shown that including a gradual build-up of mass makes little difference to the outcome of migration.

After running the model a number of times, the basic output is a plot of the final radius of a planet as a function of the time when it formed. To convert this to the number of planets expected at different radii, which is the observable quantity, we also need to specify how the probability of massive planet formation, per unit time, varies with time. We assume that this probability is uniform (or equivalently, that the rate of massive planet formation, averaged over many stars, is constant). This cannot be true over long periods of time. However, we are only interested in the window of formation times which allow a massive planet to survive migration. This window, shown as the shaded region in Figure 1, is short compared to the disc lifetime, and as a result there is only a relatively small change in the physical properties of the disc during this time. For example the disc mass, which drops by two orders of magnitude over the first 3.5 Myr, changes by less than 50%. Whatever factors influence the probability of planet formation, it should therefore be a reasonable first approximation to assume that the probability is constant over the short range of formation times which lead to a surviving massive planet.

2.4 Model for planetary migration

Our model for planetary migration is almost identical to that of Trilling et al. (2002), who describe in detail the approximations involved. The coupled evolution of the planet and the protoplanetary disc is described by the equation (Lin & Papaloizou 1986),

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[3R^{1/2}\frac{\partial}{\partial R}\left(\nu \Sigma R^{1/2}\right) - \frac{2\Lambda \Sigma R^{3/2}}{(GM_*)^{1/2}}\right]. \quad (5)$$

The first term on the right-hand side of this equation describes the diffusive evolution of the surface density due to internal viscous torques (e.g. Pringle 1981). The second term describes how the disc responds to the planetary torque, $\Lambda(R, a)$, where this function is the rate of angular momentum transfer per unit mass from the planet to the disc. We take,

$$\Lambda = -\frac{q^2 GM_*}{2R}\left(\frac{R}{\Delta_p}\right)^4 \quad R < a$$

$$\Lambda = \frac{q^2 GM_*}{2R}\left(\frac{a}{\Delta_p}\right)^4 \quad R > a \quad (6)$$

where $q = M_p/M_*$,

$$\Delta_p = \max(H, |R - a|), \quad (7)$$

and H is the scale height of the disc. Guided by detailed protoplanetary disc models (Bell et al. 1997), we adopt $H = 0.05R$. This form for Λ (equation 6) is that used by Lin & Papaloizou (1986), modified to give a symmetric treatment for the disc inside and outside the orbit of the planet.

The transfer of angular momentum leads to orbital migration of the planet at a rate,

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\left(\frac{a}{GM_*}\right)^{1/2}\left(\frac{4\pi}{M_p}\right)\int_{R_{\rm in}}^{R_{\rm out}} R\Lambda\Sigma \mathrm{d}R. \quad (8)$$

Although simple, this formalism for treating the planet-disc interaction has been shown (Trilling et al. 1998) to give comparable results to more sophisticated methods (Takeuchi, Miyama & Lin 1996).

We solve equation (5) using an explicit method on a grid that is uniform in a scaled variable $\propto R^{1/2}$, with 300 grid points between $R_{\rm in}$ and $R_{\rm out}$. The stellar mass is $M_* = M_\odot$. Typically, the timestep is limited by the radial velocity in the disc gas very near the location of the planet. Evolving the system on this (small) timestep serves no useful purpose once a gap has been opened, so we reduce Λ, and thereby limit $|v_r|$, in the vicinity of the planet. The results of the migration calculations are unaffected by this modification.

3 RESULTS

3.1 Predicted semi-major axis of stranded planets

Figure 2 shows the final semi-major axis of planets as a function of the formation time, using the standard disc model of Fig. 1. Each point represents a separate run of the model. The fate of a planet depends upon when it was formed, and upon its mass. Planets that are formed too early migrate inward of 0.1 AU, the smallest radius we consider quantitatively in this study. What happens to them subsequently depends upon whether there is a stopping mechanism to halt further migration, but many of them seem likely to be consumed by the star. Planets that start to form too late can migrate outwards, because at late times the gas in the disc at 5 AU is itself moving away from the star. At still later times, it becomes impossible to form a Jupiter mass planet at all, because the dispersing disc has too little surface density (c.f. the discussion in Shu, Johnstone & Hollenbach 1993). In between these limits, there is a window of formation times, lasting for around 0.5 Myr for a $2M_J$ planet, which result in the planet being stranded at radii between 0.1 AU and 6 AU. The migration of more massive planets slows down earlier, since their larger angular momentum produces greater resistance to disc-induced migration (Syer & Clarke 1995). Hence, the window of allowable formation times is longer for more massive planets than for less massive ones.

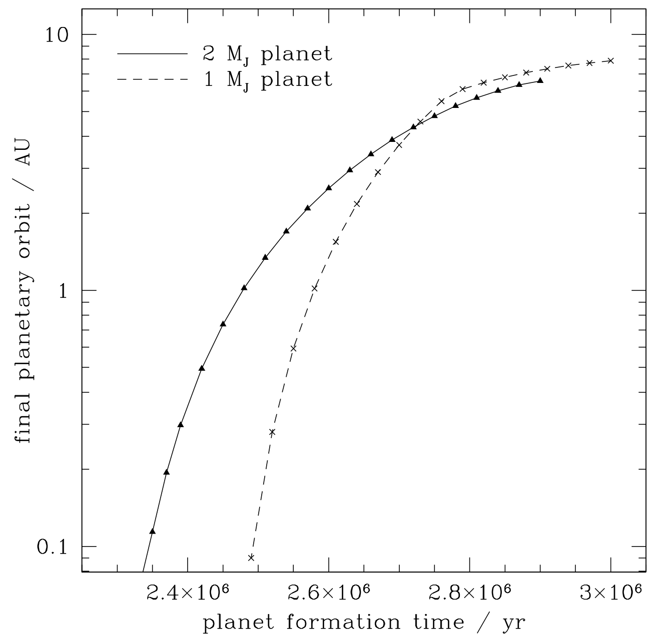

Figure 2. The final semi-major axis of planets, following migration, as a function of the planet formation time (i.e. the epoch when the planet formed at 5 AU). The standard disc model with $\beta = 3/2$ and mass loss at $R > 5$AU was used. The solid curve shows results for $2M_J$ planets, the dashed curve results for $1M_J$ planets. More massive planets can form earlier without being consumed by the star.

3.2 Dependence upon disc model and planet mass

Figure 2 shows how the final planetary radius depends upon the formation time (note that we plot $a(t_{\rm form})$ on a logarithmic scale in this figure). The curve steepens towards small radii / early formation times. This implies that if planet formation occurs with uniform probability per unit time during the allowed window, then fewer planets per logarithmic interval in a are expected at small radius (i.e. equal intervals in $\log(a)$ correspond to smaller intervals in $\Delta t_{\rm form}$ at small a).

With the assumption of a uniform rate of planet formation, the predicted number of planets per logarithmic interval in radius is,

$$\frac{\mathrm{d}N_p}{\mathrm{d}\log(a)} \propto \left(\frac{\mathrm{d}\log(a)}{\mathrm{d}t_{\rm form}}\right)^{-1}, \quad (9)$$

which can readily be derived from Fig. 2. Figure 3 shows the results for two different planet masses, and for two variants on the standard disc model. A cubic spline fit has been used

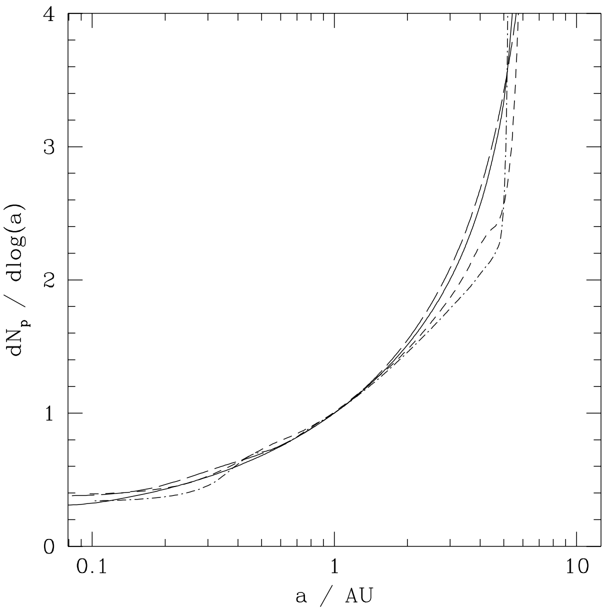

Figure 3. Predicted number of extrasolar planets per logarithmic interval in semi-major axis. These curves are derived from the $a(t_{\rm form})$ results shown in Fig. 3 by making the additional assumption that the rate of planet formation is constant. The solid curve shows results for the standard disc model ($2M_J$ planets, $\beta = 3/2$, $R_g = 5$AU). The remaining curves show the effect of changes to these parameters: a lower planet mass $M_p = 1M_J$ (short dashes), increased radius of mass loss $R_g = 10$AU (long dashes), and different viscosity $\beta = 1/2$ (dot-dashed). Because the $\beta = 1/2$ model has a lower surface density at 5 AU, the planet mass used was reduced to $0.5M_J$. The absolute number of planets is arbitrary, and has been normalised to unity at 1 AU.

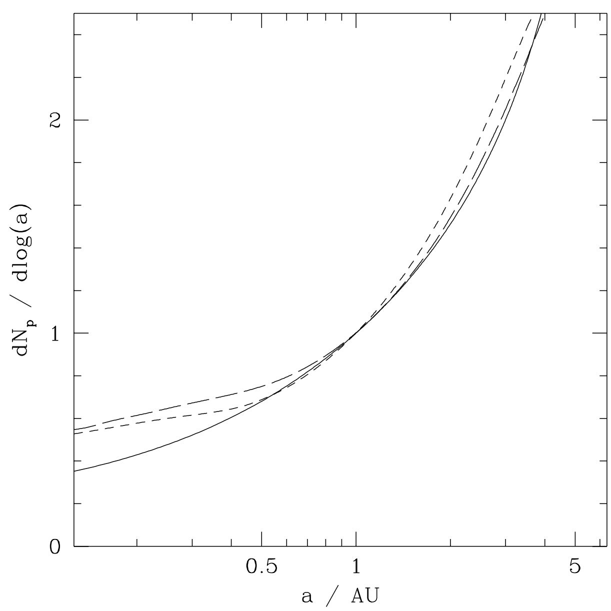

Figure 4. Predicted number of extrasolar planets per logarithmic interval in semi-major axis. The solid line shows the results for the standard disc model, as in Fig. 3. The short-dashed and long-dashed curves show models in which the disc is lost instantaneously at $t = 3$ Myr, or $t = 4$ Myr, respectively. The predicted distribution of planets that have migrated *inwards* is almost the same in all of these models.

to obtain a smooth estimate of the derivative in equation (9), and the curves have been normalised to unity at 1 AU.

The predicted number of planets per logarithmic interval in a rises rapidly with increasing radius for all the models considered. For the standard disc model, the predicted number rises by more than a factor of two between 0.1 AU and 1 AU, and by a further factor of two by 3 AU. The good agreement between the curves for $1M_J$ and $2M_J$ planets shows that the *observed* mass function for massive planets is predicted to be the same at different radii. This means that we can use the observed mass function at small radii, which is complete down to lower masses, to estimate how many low mass planets must accompany known high mass planets at larger radii. It does not mean that the initial mass function for planets is the same as the observed one. In fact, Fig. 2 makes it clear that high mass planets can form over a longer interval than low mass planets and still survive migration. To end up with the same number of $1M_J$ as $2M_J$ planets per unit interval in $\log(a)$, more low mass planets must have formed. In other words, the (currently unobservable) initial mass function for planets must rise more steeply towards low masses than the observed one.

Figure 3 also shows the predicted relative abundance of planets for two variants of the disc model. Increasing the inner radius beyond which mass is lost to 10 AU does not alter the predicted distribution of planets within 5 AU. Likewise, the predicted distribution is not substantially changed in a model where the power-law slope of the viscosity is $\beta = 1/2$. Note that for the $\beta = 1/2$ run we considered planets with mass 0.5 M_J, since insufficient gas was available between $0.6a$ and $1.6a$ to form planets of several Jupiter masses.

3.3 Comparison with an instantaneous disc dispersal model

In Figure 4, we show how the model including disc mass loss compares to a simpler model in which the disc is assumed to be lost instantaneously at a time $t_{\rm disperse}$. The disc model used is identical to our standard model with $\beta = 3/2$, except that the rate of mass loss is set to zero. We compute two models, with $t_{\rm disperse}$ equal to 3 Myr and 4 Myr respectively.

From Fig. 4, it can be seen that the instantaneous disc dispersal model makes very similar predictions for the radial distribution of inwardly migrating planets. Larger differences, however, occur for outward migration. In the instantaneous disc dispersal model, there is no outward migration of planets, because the radial velocity in the disc at 5 AU remains inward at $t_{\rm disperse}$. The model including mass loss, conversely, allows a substantial fraction of planets to migrate outwards. From Fig. 2, we find that for Jupiter mass planets as many as a third may end up in orbits exterior to that in which they formed.

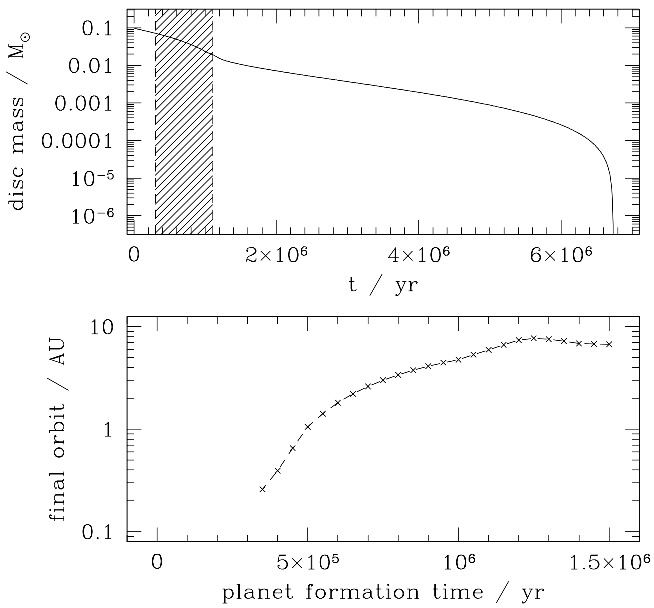

Figure 5. Results of planetary migration in a magnetically layered disc, where we have assumed that the layer is established at $t = 1$ Myr. The upper panel shows the disc mass as a function of time, with the shaded band illustrating the range of planet formation times that result in the planet having a final semi-major axis between 0.1 AU and 6 AU. The lower panel shows the final semi-major axis as a function of formation time.

3.4 Effect of ongoing planetary accretion

Even after a gap has been opened, numerical simulations show that planetary accretion may continue (Bryden at al. 1999; Kley 1999; Lubow, Seibert, & Artymowicz 1999; Nelson et al. 2000). Significant ongoing accretion – which is not included in our models – would lead to a correlation between planet mass and orbital radius, with more massive planets expected at small orbital radius. There is no evidence for such a correlation in the data, which could either indicate that migration has not occurred, or that ongoing accretion occurred only at a low level.

Lubow, Seibert, & Artymowicz (1999) find that, for low masses, the rate of ongoing accretion can be comparable to the *disc* accretion rate outside the planet. This is of the order of $10^{-9}\ M_\odot \mathrm{yr}^{-1}$ at the epoch when surviving planets form. Migration from 5 AU takes a few hundred thousand years, so we estimate that accretion across the gap could add a few tenths of a Jupiter mass to low mass planets. This is a relatively small fraction of the mass for the $2M_J$ planets that are the main focus of this paper. However, it is only marginally consistent with the existence of the lowest mass hot Jupiters, which have $M_p \sin(i) < 0.3\ M_J$. This may indicate that these planets formed within the snow line (leading to a shorter migration time).

3.5 Magnetically layered protoplanetary discs

Figure 5 shows the results for the runs using the magnetically layered disc model. The decline of the disc mass with time shows the effect of the switch to a low viscosity state. After the initial fully viscous phase, which resembles the standard model, there is a long plateau which arises because the low viscosity in the inner disc produces a bottleneck to accretion onto the star. During this period the disc is slowly destroyed from the outside inwards by the disc wind, which finally disperses the disc completely after 6 Myr. We note that, in a magnetically layered disc model, high rates of mass loss from the disc are needed in order to disperse the disc within a reasonable time-scale. The mass loss rates inferred for discs in parts of the Orion nebula are easily large enough (Johnstone, Hollenbach & Bally 1998), but it is unknown whether high enough rates are possible for isolated discs.

Figure 5 also shows the final location of migrating planets. As before, we assume these planets all formed at 5 AU, and vary the time of formation in a series of runs. The resulting $a(t_{\rm form})$ curve flattens for $a \gtrsim 1$AU. As with the standard disc model, therefore, we predict a broad distribution of final orbital radii, with more planets (per logarithmic interval in a) ending up at large radii than at small radii. However, there are also important differences. Planets must form *earlier* (relative to the disc lifetime) in the layered disc if they are to migrate successfully to small orbital radii. This is because little migration can take place once the disc has entered its quiescent, low viscosity phase. For a planet to end up inside 1 AU, it must have formed at an earlier time when the disc was still viscous and able to drive rapid migration.

4 COMPARISON WITH CURRENT DATA

Before comparing our results to current data, we need to consider possible biases. In an idealized radial velocity search for extrasolar planets, two important selection effects are expected. First, there is almost no sensitivity to planets with orbital periods P longer than the duration of the survey (until a whole orbit is seen, the planetary signal may be confused with that from a more massive, distant companion). Second, a planet with mass M_p and orbit inclination i produces a radial velocity signal with an amplitude proportional to $M_p \sin(i) a^{-1/2}$. A survey which makes a fixed number of observations, with some given sensitivity, can therefore detect planets above a minimum $M_p \sin(i) \propto a^{1/2}$. Although this is a relatively weak radial dependence, it still corresponds to an order of magnitude difference in the minimum detectable mass over the range of radii probed by current surveys.

Real surveys are less easily characterised. Instruments have improved steadily over time, and the intrinsic limits to radial velocity measurements vary on a star-by-star basis. Nevertheless, the selection effect caused by the scaling of the minimum detectable mass with $a^{1/2}$ can readily be seen in the data. Figure 6 shows the location of all the known extrasolar planets with $M_p \sin(i) < 10\ M_J$, in the $M_p \sin(i)$ - a plane. Most of the hot Jupiters at $a < 0.1$ AU, where surveys are most sensitive, have masses that are actually substantially smaller than that of Jupiter, while the typical detected planet with an orbital radius greater than 1 AU has a mass of several M_J.

A simple count of the total number of detected planets as a function of radius leads to the cumulative distribution

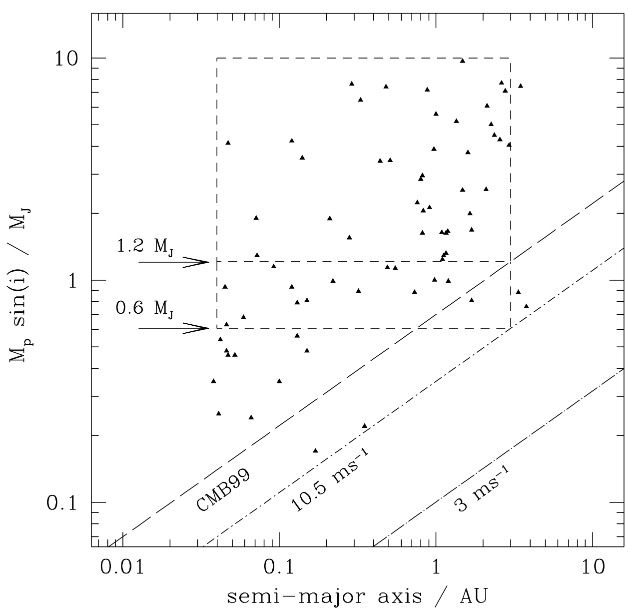

Figure 6. The observed distribution of extrasolar planets in the $M_p \sin(i)$ - a plane. The diagonal lines indicate the approximate selection limits of current radial velocity surveys. The upper line, labelled CMB99, shows the minimum mass above which planets are detectable at 99% confidence, according to the analysis of Cumming, Marcy & Butler (1999). We also plot lines corresponding to radial velocity amplitudes of 3 ms^{-1} and 10.5 ms^{-1}. The rectangular boxes define subsamples of the data, used for comparison with the predictions of disc migration models.

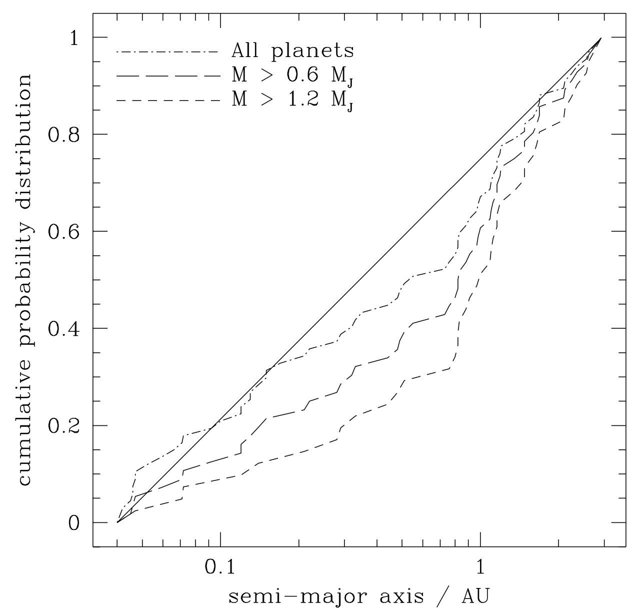

Figure 7. The cumulative probability distribution of extrasolar planets as a function of orbital radius. The dot-dashed curve shows the distribution for all detected planets. This distribution is consistent (KS test probability $P = 0.05$) with a uniform distribution in $\log(a)$, shown as the solid line. We also consider subsamples comprising only those planets with masses above 0.6 M_J (long dashes), or 1.2 M_J (short dashes). The hypothesis that these subsamples have a distribution that is uniform in $\log(a)$ can be rejected at more than 99.9% confidence.

	$M_p \sin(i) > 1.2M_J$	$M_p \sin(i) > 0.6M_J$
0.040 − 3.0 AU	5×10^{-6}	7×10^{-4}
0.065 − 1.9 AU	10^{-4}	10^{-3}

Table 1. Kolmogorov-Smirnov test probability that the observed distribution of extrasolar planets is drawn from a distribution uniform in $\log(a)$. The subsamples considered have masses exceeding 0.6 M_J or 1.2 M_J, and orbital radii within the two quoted ranges.

shown in Figure 7. A Kolmogorov-Smirnov test (e.g. Press et al. 1989) shows that this is marginally consistent (KS probability $P = 0.05$) with a uniform distribution in $\log(a)$. However, from Fig. 6, it can be seen that this uncorrected distribution includes low mass planets, at small orbital radius, that would not be detectable beyond 1 AU. The mass function of planets has substantial numbers of low mass planets (Zucker & Mazeh 2001; Tabachnik & Tremaine 2002), so this leads to a significant selection effect (Lineweaver & Grether 2002).

An unbiased estimate of the radial distribution of extrasolar planets can be obtained by considering a subsample of planets which are sufficiently massive that they could be detected at *any* radius. A conservative definition of such a subsample can be obtained by noting that for the Lick planet search, circa 1999, the minimum detectable mass at 99% confidence is quoted by Cumming, Marcy & Butler (1999) as,

$$M_p \sin(i) \gtrsim 0.7 M_J (a/AU)^{1/2}. \quad (10)$$

Taking this as a guide, a mass cut at $M_{\rm cut} = M_p \sin(i) > 1.2\ M_J$ should yield an approximately complete subsample out to 3 AU. Since the precision of the observations is likely to have improved since 1999, we also consider a subsample with the mass cut placed at 0.6 M_J. This corresponds to a radial velocity amplitude at 3 AU of 10.5 ms^{-1}, which is equal to the smallest amplitude signal of any detected planet. As one might expect, this practical detection threshold lies at $3 - 4\sigma$, where $\sigma \approx 3$ ms^{-1} is the best-case error on a single radial velocity measurement.

As shown in Fig. 7 and Table 1, neither of these subsamples is consistent with the hypothesis that it is drawn from a uniform distribution in $\log(a)$, which can be excluded at the 99.9% confidence level. This conclusion is also robust to different choices of the minimum and maximum orbital radii of planets in the subsample.

To compare with the theoretical distribution derived earlier, we henceforth take $M_{\rm cut} = 0.6\ M_J$. This may somewhat *underestimate* the number of planets at radii of 2-3 AU, but any such bias is smaller than the uncertainties introduced by the relatively small sample size. We then take the current compilation of 76 detected planets (from `exoplanets.org`), discard those with masses outside the range $0.6M_J < M_p \sin(i) < 10M_J$, and bin 54 survivors into four logarithmic intervals in semi-major axis, 0.033 AU – 0.1 AU, 0.1 AU – 0.3 AU, 0.3 AU – 0.9 AU, and 0.9 AU – 2.7 AU.

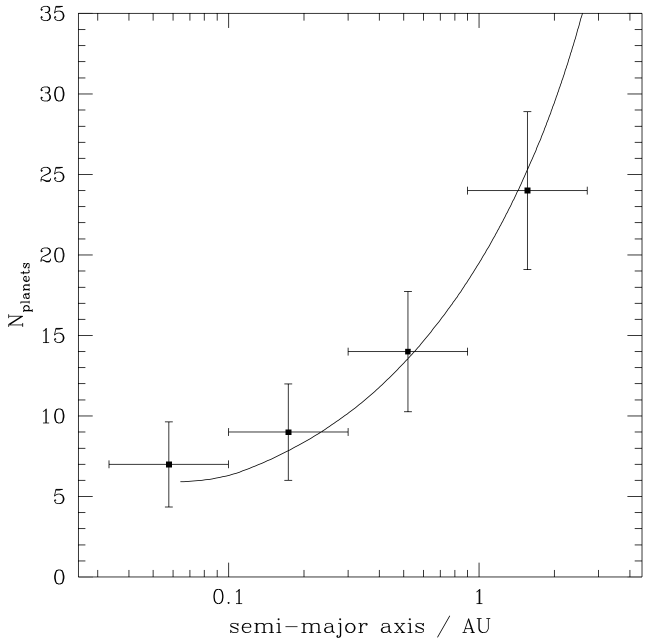

Figure 8. Predicted and observed number of extrasolar planets as a function of radius. The data (points with error bars showing $\sqrt{N}$ errors) show the number of extrasolar planets in four radial bins, using a subsample with a mass cut at 0.6 M_J. The theoretical curve is the prediction for planetary migration within the standard disc model, with the normalisation chosen to give the best fit to the data.

Figure 8 shows the comparison between observation and theory. The disc inside 0.1 AU may well be strongly influenced by the protostellar magnetosphere (Konigl 1991; Najita, Edwards, Basri & Carr 2000), which we have chosen not to model. We therefore concentrate attention on the outer three radial bins, between 0.1 AU and 2.7 AU. The number of known extrasolar planets in this (approximately) complete subsample rises rapidly with increasing semi-major axis. Although the error bars on the data are large, excellent agreement is obtained with the predictions of the theoretical migration model using the standard disc model. This agreement persists when different choices of binning and sample selection are made. In particular, we can obtain an equally good fit (with a different normalization) if we adopt a more conservative mass cut at 1.2 M_J.

From the Figure, we also note that in *this mass range*, the number of hot Jupiters at $a < 0.1$AU is in fact consistent with an extrapolation of the data from larger radii. There is no need for an additional stopping mechanism to explain the abundance of these planets at the smallest radii. There is, however, weak evidence for an over-abundance of lower mass planets (with $M_p \sin(i) < 0.6 M_J$, not plotted in the Figure) in the innermost radial bin.

5 DISCUSSION

5.1 Frequency of massive planets

The good agreement between theory and observations seen in Figure 8 allows us to use the model to estimate the fraction of stars targeted by radial velocity surveys that possess massive planets, including those with masses too low, or semi-major axis too large, to have been detected so far. Of course, we cannot extrapolate to masses smaller than the $\sim 0.2 M_J \sin(i)$ that is the smallest yet detected, as we have no information at all on the mass function below this threshold. We can, however, estimate how many planets with $0.5 M_J \sin(i)$ ought to exist at 1 AU (say), beyond the radius where they are currently detectable.

To do so, we note that the radial velocity surveys are most complete (i.e. reach down to the smallest masses) at small radii. Between 0.1 AU and 0.2 AU, equation (10) suggests that planets with masses of $0.3 M_J \sin(i)$ and above are detectable. 8 planets are known in this radial interval, corresponding to approximately 1% of target stars (since the total haul of 76 planets implies that approximately 8% of stars have planets). The range of planet formation times which result in the planet becoming stranded between 0.1 AU and 0.2 AU is approximately 2.5×10^4yr, which may be compared to the 0.42 Myr window which leaves planets between 0.1 AU and 5 AU. Hence, we estimate that the fraction of target stars possessing planets with masses $0.3 M_{\rm J} < M_p \sin(i) < 10 M_{\rm J}$, and radii $0.1{\rm AU} < a < 5{\rm AU}$, is ≈ 15 %. Roughly half of these have already been detected.

More speculatively, we can also try to extrapolate to larger radii. If we assume that massive planets at 5-8 AU formed near the inner boundary of that region and migrated outwards, then a similar analysis to that above suggests that an additional 5-10% of stars could possess such planets. This conclusion is more tentative than for inward migration, however, because the extent of outward migration varies significantly with planet mass (Fig. 2), and could be very different in the magnetically layered disc model.

5.2 Frequency of massive planet formation

In our standard model, planets which survive migration with final orbital radii between 0.1 AU and 6 AU must have formed during a 0.5 Myr allowed window. This window lasts for only $\approx 20\%$ of the disc lifetime. Migration without a stopping mechanism is thus a moderately inefficient process (Trilling et al. 2002) – several planets are likely to have formed for every planet which survives today. This inefficiency means that either a large fraction of stars formed of the order of one planet, or that a smaller fraction of stars formed many planets. Our models do not distinguish between these possibilities, though in principle they make different predictions. For example, the fraction of stars which consume planets (and may show resulting metallicity enhancements) is smaller if only a few stars form many massive planets. The interpretation of current metallicity measurements within such a framework, however, remains somewhat controversial (e.g. Gonzalez 1997; Udry, Mayor & Queloz 2001; Murray et al. 2001; Pinsonneault, DePoy & Coffee 2001; Suchkov & Schultz 2001).

5.3 Number of planets detectable via microlensing

Detailed monitoring of microlensing events has so far failed to find anomalies in the light curves characteristic of massive, bound planets (Bennett & Rhie 1996; Albrow et al.

2001; Gaudi et al. 2002). The results exclude the possibility that more than around a third of lensing stars have Jupiter mass planets at radii 1.5AU $< a <$ 4AU. Our results are consistent with this limit. For masses greater than $0.3 M_J$, we estimate that 7% of the stars targeted by radial velocity surveys have planets within this range of radii. If this is also true of the lensing stars (which are typically less massive – around $0.3 M_\odot$), then it suggests that the existing monitoring campaigns are within a factor of ~ 5 of reaching limits where a detection may be expected.

5.4 Probability of forming a planet which does not migrate

Unlike in the case of extrasolar planetary systems, the evidence for migration in the Solar System is extremely limited. *In situ* analysis of the Jovian atmosphere shows that it may contain material that originated in the outer Solar System (Owen et al. 1999), while substantial migration of Uranus and Neptune has been suggested (Thommes, Duncan & Levison 1999). There is little to suggest that Jupiter formed a significant distance away from its current location.

Migration models permit both inward and outward migration, so forming a planet which does not migrate is perfectly possible, though it requires fortuitous timing. If we take 'no significant migration' to mean that the planet moves a distance,

$$\frac{\Delta a}{a} \leq 0.1, \tag{11}$$

then in our standard disc model the window of formation time over which this occurs lasts for $\Delta t = 6.8 \times 10^4$yr. This may be compared with the 0.5 Myr range of formation times that result in the planet having a semi-major axis in the range 0.1AU $< a <$ 6AU. Making the same assumption as before – that the rate of planet formation remains constant across this larger interval – we estimate that no significant migration occurs in approximately 10-15% of systems in which a planet survives. The no migration outcome which may describe Jupiter in the Solar System would then be an unlikely event, but not a rare one (Lineweaver & Grether 2002). Of course forming *several* planets, none of which migrated significantly, would be much less likely. However, as already noted, migration of the other giant planets in the Solar System has been seriously considered and does not appear to be excluded on observational grounds.

5.5 Eccentricity

Our model for migration ignores the typically substantial eccentricities of extrasolar planets. How serious this omission is depends upon the (unknown) mechanism which leads to the eccentricity. If interactions between a single planet and the disc are responsible (Artymowicz et al. 1991; Papaloizou, Nelson & Masset 2001; Goldreich & Sari 2002), then a description similar to the one we have developed here ought to suffice. Conversely, eccentricity may arise from planet-planet scattering in a multiple planet system (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Papaloizou & Terquem 2001). This would be accompanied by order unity changes in the semi-major axis of the surviving planet. The predicted distribution of planetary orbital radii, shown in Figure 8, would be changed if all planets (say) suffered a shrinkage of their orbits by a factor of two subsequent to migration. We have no additional need to invoke such a process to explain the orbital radii of the planets, but more work in this area is warranted.

6 CONCLUSIONS

We have shown that the radial distribution of massive extrasolar planets, beyond 0.1 AU, is consistent with the planets forming at 5 AU, before migrating inwards through a viscous protoplanetary disc. This means that although theory suggests that massive planets could form closer to the star (Papaloizou & Terquem 1999; Bodenheimer, Hubickyj & Lissauer 2000; Sasselov & Lecar 2000), there is no *observational* requirement for planets to form at $a <$ 3AU. The predicted distribution is a robust feature of disc models which include dispersal of the gas via a disc wind, so the level of comparison is currently limited by the small numbers of detected planets. The continuation of existing radial velocity surveys, along with forthcoming astrometric searches (e.g. Lattanzi et al. 2000), will allow for more stringent tests, provided that the selection effects of the surveys are well understood.

The good agreement between observations and theory allows us to use the model to estimate the fraction of stars that possess massive planets, including those currently undetected on account of their low mass or large semi-major axis. Limiting ourselves to planets with masses $0.3 M_{\rm J} < M_p \sin(i) < 10 M_{\rm J}$, and radii 0.1AU $< a <$ 5AU, we estimate that around 15% of stars in the current target sample of extrasolar planet searches possess planets. In some of our models a significant number of planets migrate *outwards*, and these could form a sizeable additional population. These numbers are comparable to those found by Trilling, Lunine & Benz (2002), and imply that even in existing samples, most of the massive planets remain to be discovered.

Finally, we have used the results to quantify the fraction of systems in which the most massive planet does not suffer significant radial migration. This allows us to address the concern that migration theories could be inconsistent with the detailed knowledge of our own Solar System, in which Jupiter may have formed close to its current location. We find that a no-migration outcome requires fortuitous timing of the epoch of planet formation, but not extraordinary luck. Of the order of 10-15% of planetary systems are expected to have a massive planet which has migrated by less than 10% in orbital radius. Having a most massive planet at the orbital radius of Jupiter is thus expected to occur in around 1-2% of solar-type stars. The orbital radius of Jupiter, at least, is consistent with our understanding of extrasolar planets, though the near-circular orbit may yet prove to be unusual.

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